Symmetry: Culture and Science
Vol. 12, Nos. 1-2, 53-65, 2001

ISLAMIC GEOMETRICAL PATTERNS FOR THE TEACHING OF MATHEMATICS OF SYMMETRY

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Abstract: *From the 10^{th} to the 13^{th} century AD, aided by mathematicians (Ozdural 2000), the artists and artisans in the Islamic civilization produced a large body of symmetric geometrical patterns. Many of these are extremely elegant and convey something deeply profound about the beauty of geometrical forms which cannot be put into words.*

Islamic patterns may of course be enjoyed purely as decorations. They can be enjoyed for aesthetic experience of art and science in unity. But these patterns depict a variety of geometrical structures and constraints of the Euclidean space. Hence they are of great merit as educational aids for the teaching of many topics in mathematics, physics, chemistry, crystallography, computer science and design. In particular they can be valuable in the teaching of geometry to school children and provide a visual gateway for the teaching of abstract notions of Group Theory at the university level.

This paper will introduce Islamic geometrical patterns, say something about their characteristics and origins and point the reader to a variety of resources on Islamic Patterns (books, papers, videos and websites) which may be utilised in the teaching of the mathematics of symmetry.

1. INTRODUCTION

Geometric patterns occur in rich profusion throughout Islamic cultures. They are found on a diversity of materials – tiles, bricks, wood, brass, paper, plaster, glass and on many types of objects. They occur on carpets, windows, doors, screens, railings, bowls,

furniture-specially pulpits in mosques, and on other surfaces. They can be seen in abundance in Persian miniatures in the illumination of the Holy Koran and on architectural surfaces of mosques, palaces, madersas (centres of education) and tombs.

Symmetric Islamic patterns come in three distinct geometrical flavours. One of these is that of *Calligraphic* patterns which most often mould Arabic lettering for words such as *Allah* and *Mohammed* or short verses from the Koran, to create symmetric geometrical forms. Figure 1 shows an example Calligraphic Pattern which uses the word *Mohammed.* It is from the tomb of the Sufi poet Shah Abdullah Ansari in Hirat, Afghanistan.

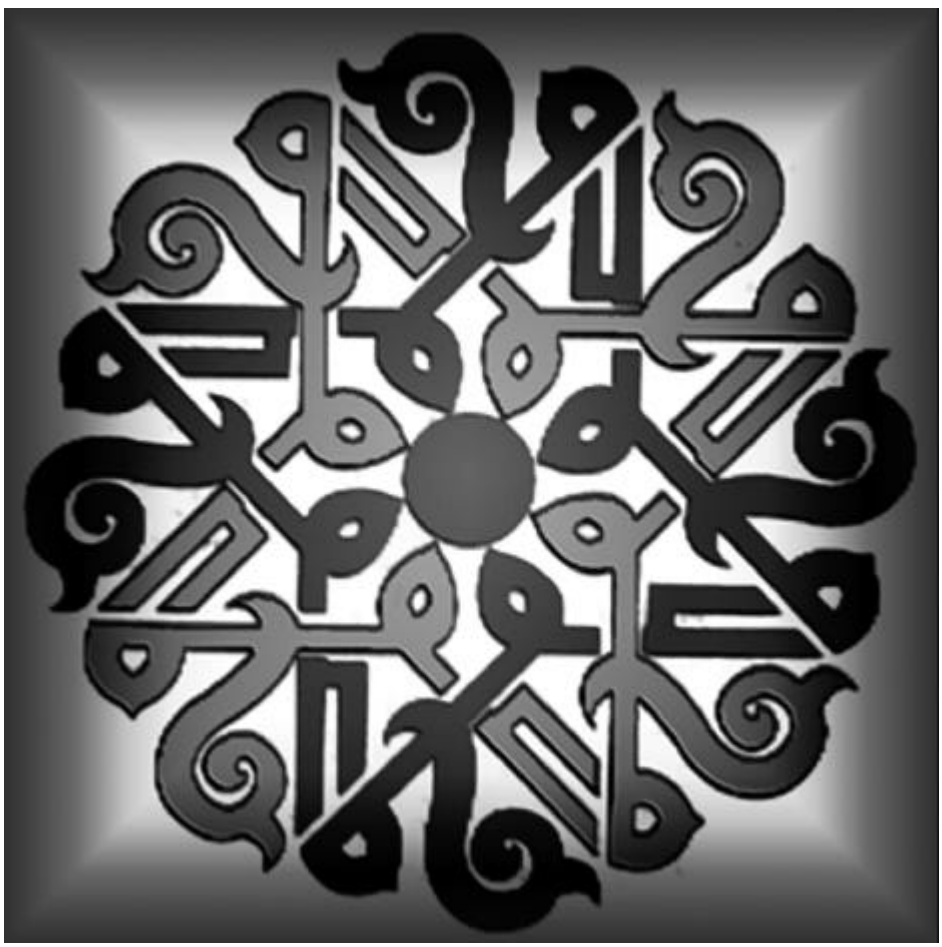

Figure 1

A second distinct pattern type perfected in Islamic art is the *Arabesque*. In such patterns spiral forms intertwine, undulate and coalesce rhythmically to produce stylized leaves and floral forms. An example is shown in Figure 2.

Figure 2

The third and largest class of Islamic patterns employs polygons, and less frequently, regions bounded by circular arcs, to produce *space filling patterns*. When such patterns are rendered on a two-dimensional flat surface a basic unit cell repeats itself over and over again (Figures 3-9). A consequence of this is that there is no natural point of focus for the eye. As one looks at an expanse of pattern the eye 'flows' continuously following the lines and seeing a variety of intricate structures and relationships. In three dimensions, such as interior and exterior surfaces of domes, the unit cell is skillfully scaled and deformed to fit the surface (Figure 10).

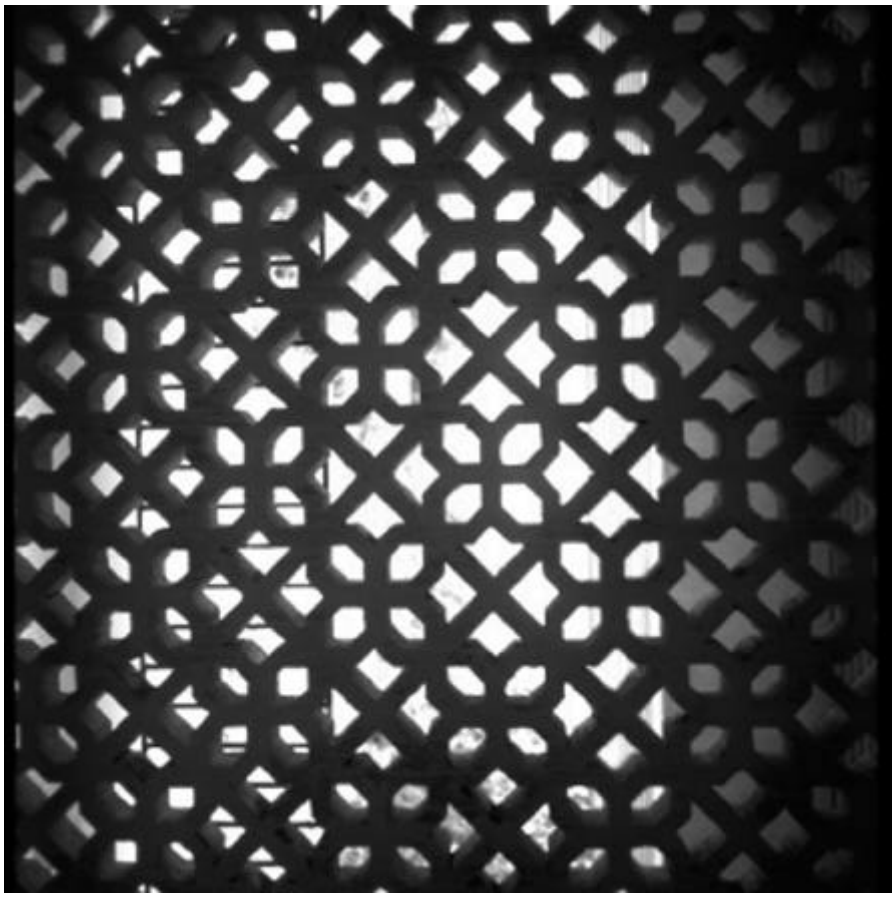

Figure 3

2. CHARACTERISTICS OF ISLAMIC PATTERNS

Using Figures 3-9, we shall now point out the characteristics of Islamic Patterns. The pattern shown in Figure 3 has a very simple structure. It is easy to see that it has been created by placing 4 identical hexagons to form petal shapes inside a square unit cell. This pattern does not display the characteristic shapes that occur in the majority of Islamic patterns. It has been included to make the point that Islamic patterns occur in many shapes and some rather simple ones do not possess the instantly recognizable Islamic flavour, which is displayed by the vast bulk of the more complex patterns. The patterns shown in Figures 4-11 are, in contrast, all instantly recognizable as being 'Islamic'.

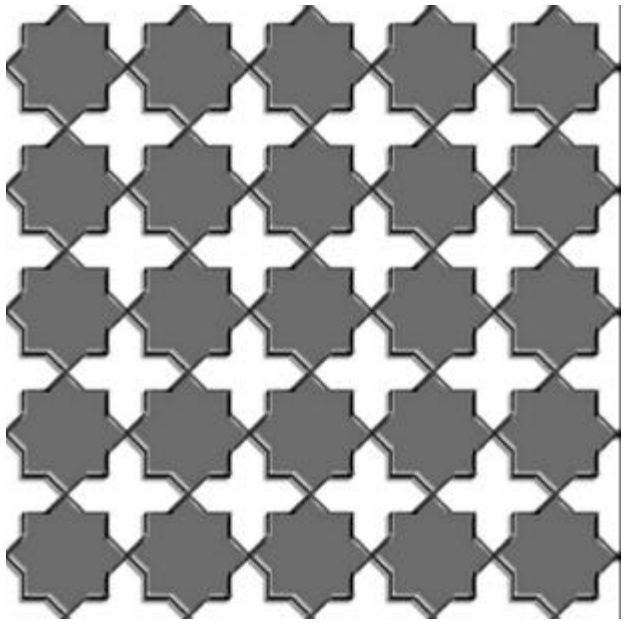

Figure 4

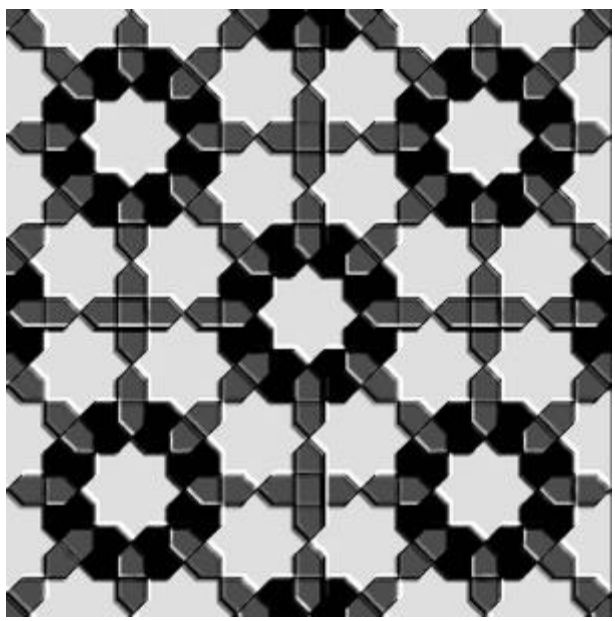

Figure 5

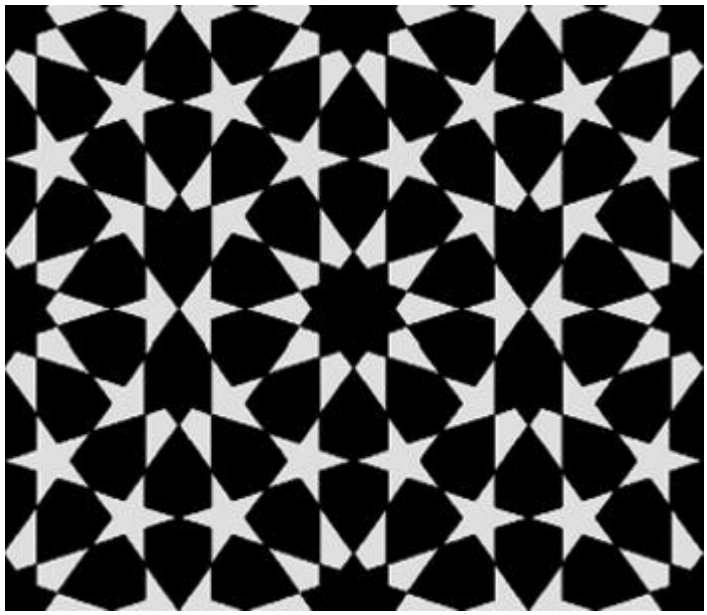

Figure 6

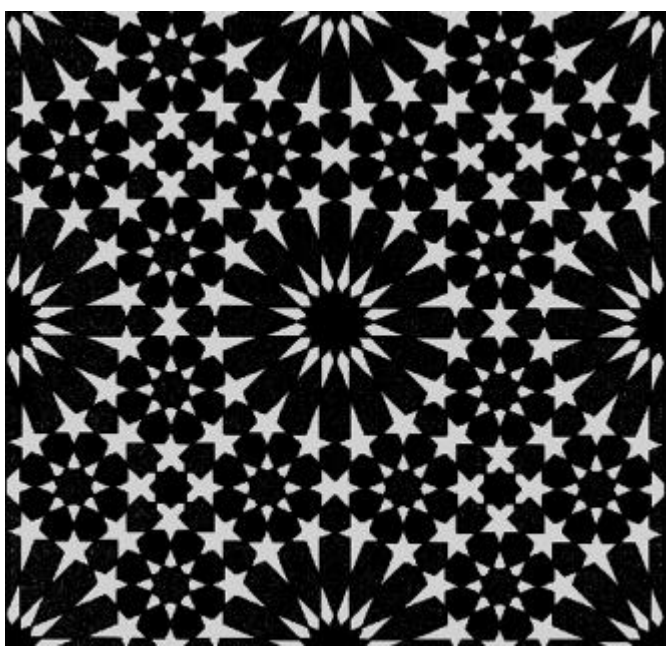

Figure 7

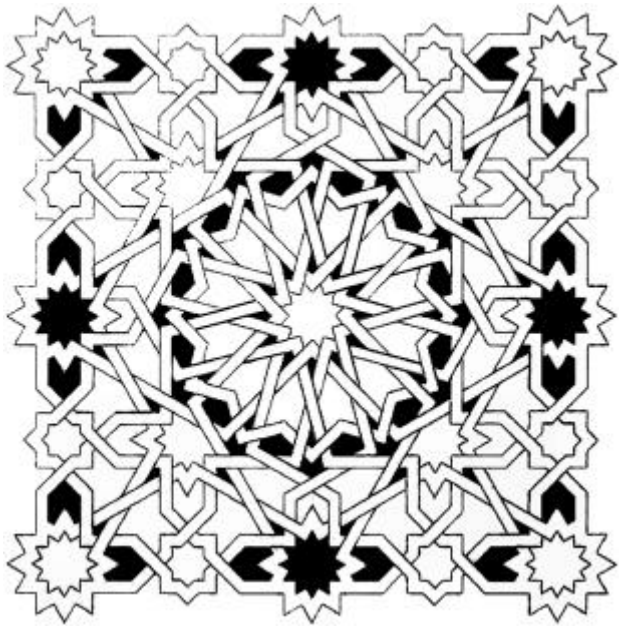

Figure 8

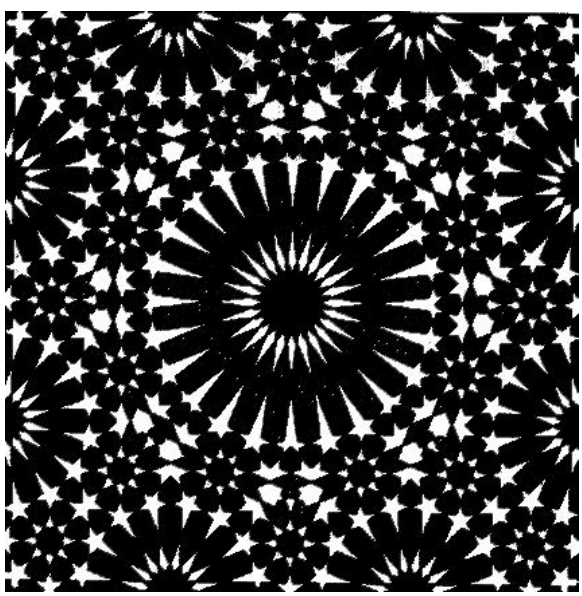

Figure 9

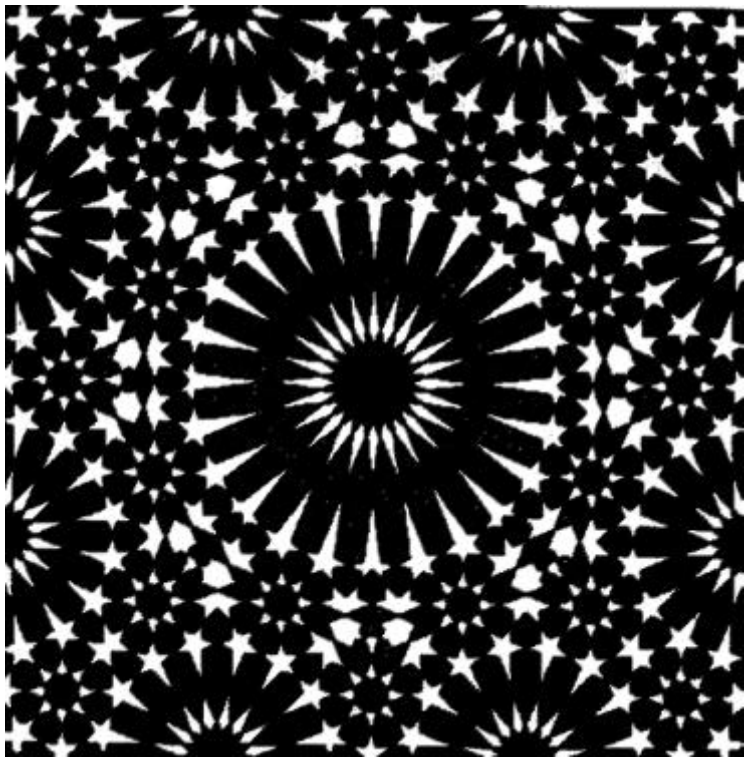

Figure 10

Figure 11

The most striking characteristic of Islamic geometrical patterns (Figures 4-11) is the prominence of star and rosette shapes. Such shapes with five, six, eight, ten, twelve and sixteen rays are the ones that occur most frequently, but patterns containing other number, particularly in multiples of eight up to ninety six, can be found.

3. THE PSYCHOLOGY BEHIND SYMMETRIC STAR-SHAPED ISLAMIC PATTERNS

It is often said that geometry was forced on Islamic art because Islam forbids the drawing of animate shapes. In the view of the author, this explanation suffices only at a rather superficial level and misses out deeper and more significant reasons.

The first thing that needs to be said to challenge this simplistic repetition is that there is a vast body of figurative work by Muslim artists. Everyone has seen examples of Persian miniatures, but apart from these, there exist a large number of realistic life-like pictures of humans as well as animals executed with great virtuosity and naturalism by Muslim artists. This is especially true for work produced during the Moghal period in India, but such works exist in other places as well.

The Moorish artists, for example, also produced a whole variety of pictures of animate objects and there is a fine set of portraits, for all to see, of the kings of Alhambra on the ceiling in the *Hall of Kings* in the Alhambra. The author has challenged the prohibition based explanation at length in the References (Abas and Salman 1995) and here a summary will be given.

In the view of the author the major influences, which forged Islamic geometric art, are as follows:

(i) Islam Has No Image of God Except Light and Stars Radiate Light. The Wandering Nomads In Deserts Also Rely on Stars for Navigation

Unlike, say, Christianity and Buddhism, Islam offered no image of God.

The only material image of God that the Koran offers is that of *Nur*, meaning light. “God is the light of the heavens and earth”, it proclaims. Since stars produce the light of heavens, *it is not at all surprising that Muslim artist should produce art containing star shapes for sacred buildings such as mosques and tombs and in the illumination of the Koran.*

There were also strong practical reasons for the passion for stars.

The Arabs who produced Islam dwelt in deserts and their way of life involved nomadic wandering over large areas of land. They were also seafarers and sailed over considerable distances. The navigation in both kinds of travel demanded skilful observation of the heavens.

Furthermore, Islam enforced a unique requirement on the faithful. Whether on land or sea a Muslim has to know, five times a day, the exact direction in which to pray. All this made the stars extraordinarily significant to the early Islamic cultures. The Koran abounds with verses which conjure up powerful imageries on the theme – "Allah it is who hath set for you the stars that ye may guide your course by them amid the darkness of the land and the sea" (V: 98). This then is yet another reason for the prominence of star shapes in Islamic art.

(ii) Geometry is the Door from the Material World to the Spiritual World and the Glimpse of Perfection

Long before the birth of Islam, several of the classical Greek philosophers had associated metaphysical qualities with geometry. The abstract definitions and logical consistency of the subject had been seen as pointers to a perfect world underlying gross reality and hence to the perfection of gods. "God ever geometrises", Plato had proclaimed.

Imbued with the idea of an abstract God, Muslim intellectuals found the notions of Greek geometers immensely agreeable and concurred that geometry offers the unifying intermediary between the material and the spiritual world. It is not surprising that Muslim artists should seek to depict perfection through geometry.

(iii) Long Experience of Carpet Weaving Gave Tent Dwellers Skill and Passion for Tessellations, Interlaced Patterns and the All-Over Covering of Surfaces

Carpets and rugs are the most natural form of furniture for tent dwellers. They have been produced throughout the Middle East and the Caucasus region for a very long time. In particular, the nomadic tribes of Central Asia, Persia and Afghanistan have been producing carpets and rugs for thousands of years. They serve many purposes; as floor coverings, prayer mats, tent decorations, canopies, as symbols of power, privilege and riches. Carpets represented the most ancient and the most meaningful art form in the population that first embraced Islam.

The creation of this art involves interlacing to produce tessellating repeat patterns. It is not surprising that Islamic buildings reflect this long tradition.

Thus, we see that there are very sound positive reasons why Islamic art should naturally turn to geometry, star shapes, tessellations and interlaced patterns and not simply the negative fear of hell fire, which is regularly suggested.

3. THE USE OF ISLAMIC PATTERNS IN TEACHING THE MATHEMATICS OF SYMMETRY

Islamic patterns provide a vast ready-made stock of material for the teaching of symmetry at all levels – from kindergarten to university. We shall not discuss the matter in any great detail here, but make some general comments to justify what has just been said. After that we shall refer the reader to variety of resources, where suitable material can be found to develop the subject in detail.

Children, from a very early age, love colouring patterns and Islamic patterns may be utilised introduce symmetry at the tender age of 3 or 4. At the university level, the patterns may be used to teach transformation geometry. The mathematical process involved in the creation of these patterns involves the application of symmetry transformations (rotations, reflections, translations and glide reflections) on a two-dimensional template motif. These transformations form a group and hence these patterns provide a visual gateway to learning about the abstract ideas of Group Theory which is the key tool in the mathematical description of the interactions of the fundamental forces of nature.

4. REFERENCES TO SOME SUITABLE RESOURCES

This last section will point the reader to a number of books, papers, websites and videos on Islamic Patterns, which may be utilised for learning about Islamic patterns and utilising them in the teaching of the mathematics of symmetry. We shall make brief comments to indicate at the level at which the resource is suitable.

4.1 Books

Gülru Necipoglu with an essay by Mohammad al-Asad (1995) *The Topkapi Scroll Geometry and Ornament in Islamic Architecture*, Getty Research Institute.

Description: The Topkapi Scroll, discovered, in 1986 at Istanbul's Topkapi Palace Museum Library, is a rare

and most valuable resource on Islamic Patterns. It is a pattern book from the workshop of a master builder who worked in Persia during the late 15th or 16th century and contains 114 drawings. It is the earliest manuscript of its kind to have been found intact.

The above book discusses and illustrates the material in the scroll and constitutes the most authoritative description of the methodology of Islamic Geometric Art.

Suitability: This work is for serious scholars of Islamic art.

Grünbaum, B. and Shephard, G. C. (1987) *Tilings and Patterns*, W. H. Freeman.

Description: This is the Bible of the mathematics of Tilings and Patterns and contains many Islamic Patterns.

Suitability: Apart from many pretty diagrams, which may delight anyone, the work is only suitable for the mathematicians.

Abas, S. J. and Salman, A. S. (1995) *Symmetries of Islamic Geometrical Patterns*, World Scientific.

Description: The most comprehensive book on symmetry analysis of Islamic Patterns.

Suitability: The first three chapters are of historical and philosophical nature and may be enjoyed by any intelligent reader. The fourth chapter is aimed at students and teachers of mathematics at university level.

Bourgoin, J. (1973*) Arabic Geometrical Pattern and Design*, Dover Publications.

Description: The first publication on a large collection of Islamic Patterns.

Suitability: All levels. Ideal as a colouring book for young children.

Teaching Maths Through Islamic Art (1993) Victoria And Albert Museum, London.

Description: Aimed at teachers of mathematics and based on Islamic Art objects to be found in the Victoria And Albert Museum, London.

Suitability: Students and teachers of mathematics at school level.

4.2 Papers

Ozdural, A. (2000) Mathematics and Arts: Connections between Theory and Practice in the Medieval Islamic World, *Historia Mathematica*, 27, 171-201.

Description: The paper explores the interactions between mathematicians and artisans in the Medieval Islamic World.

Suitability: Art Historians and those with serious interest in the origins of Islamic Patterns.

Schattschneider, D. (1978) The Plane Symmetry Groups, Their Recognition and Notation, *American Mathematical Monthly*, 85, 439-450.

Description: This is a classic paper on the notation of the Plane Symmetry Groups.

Suitability: Mathematicians, physicists, chemists, crystallographers and also those who wish to apply symmetry analysis in art.

Abas, S. J. and Salman, A. S. (1992) Geometric and group-theoretic methods for computer graphics studies of Islamic symmetric patterns, *Computer Graphics Forum* 11, 1, 43-53.

Description: An early effort to utilise computer graphics to symmetry transformations.

Suitability: Those interested in developing Computer Graphics algorithms for exploring Islamic Patterns.

4.3 Video

Costa, A.F., Mora, J., Gomez, B., U. N. E. D. (2002) *Arabesques and Geometry* Springer VideoMATH.

Description: This video, spectacularly illustrated by examples Islamic Patterns from the Alhambra in Granada, Spain, explains the topic of symmetry in mathematics with examples from Islamic art.

Suitability: All levels from school students to researchers.

4.4 Websites

Symmetry of Rugs -Field Patterns: http://mathforum.org/geometry/rugs/symmetry/fp.html

Description: An excellent site for learning about symmetry and rugs.

Suitability: All levels from school students to researchers.

*Religious Beliefs Made Visual: Geometry and Islam***,** Jane Norman, consultant, education department, Metropolitan Museum of Art, New York, http://www.askasia.org/frclasrm/lessplan/l000030.htm.

Description: Jane Norman pioneered the utilisation of Islamic Patterns in teaching mathematics to school children and this is her site.

Suitability: School teachers and students.

Islamic Art for the 21st Century, Syed Jan Abas, http://www.bangor.ac.uk/IslamicArt.

Description: Very popular site run by the author to display his work in Islamic Art.

Suitability: Everyone, especially those interested in the unity of science and art.

*Taprats: Computer-Generated Islamic Star Patterns***,** Craig S. Kaplan,

http://www.cgl.uwaterloo.ca/~csk/washington//taprats/.

Description: A beautiful site. Supplies a Java applet for generating Islamic star patterns.

Suitability: Every computer literate person will enjoy a visit to this site.