THE RELATIONSHIP BETWEEN MATHEMATICS EDUCATION AND CULTURE

by
Alan J. Bishop
Faculty of Education
Monash University,
Victoria 3168, Australia

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The relationship between mathematics education and culture.

Alan J Bishop
Faculty of Education, Monash University
Clayton, Victoria 3168, Australia

Introduction - the challenge of teaching meaningful mathematics

Mathematics is one of the most important subjects in schools in a modern society but teaching it well is very difficult. It is a subject which can become abstract very quickly and that means that for many pupils it can become meaningless as it loses contact with the real world that they know outside school. The result is that all around the world pupils are failing in their mathematics courses, and many adults don't understand it either. Many teachers also find it a difficult subject to understand.

If I say to other people that I train mathematics teachers they look at me as if I am a strange creature. If I say that I enjoy mathematics, they think I am crazy, and if I say I can also help them to enjoy it too they just don't believe it! Mathematics to the average person is to be suffered, it is a kind of torture of the mind! You would think therefore that they would like to get rid of mathematics from the school curriculum, but no! They think its very important and that all schoolchildren should study it, even if they won't enjoy it. It is good for them!

Of course we know that it is important for all pupils to study mathematics, and as educators we have to be responsible for finding a way to solve this problem? We have to look at the curriculum, the teaching, the teaching materials, and the teacher training practices for improvements. Mathematics teachers, teacher trainers, curriculum developers, textbook writers, and others involved in mathematics teaching, will be referred to in this talk as "mathematics educators", and their work has become increasingly difficult as societies have become more complex, as there are more demands to teach mathematics to all pupils, and as the mathematical content needing to be taught has also increased in complexity. So in trying to meet the challenges of the new situation, leading mathematics educators around the world have been doing a lot of new research.

In my talk today I will introduce you to some of these new ideas which we have been working on during the last twenty five years. I realise that 25 years is like "the blink of an eye" in the timescale of the proud history of Iran, but with this blink of the eye has come the realisation that we can now create a better and more meaningful mathematics education for our children than we ever could have done in the past. The ideas all arise from my development of the social dimension of mathematics education (see Bishop, 1993).

This dimension is stimulating research at several different levels, the most important of which are:

- the individual level, concerning the individual mathematics learners, both inside and outside the classroom,
- the pedagogical level, concerning the many social interactions taking place in the mathematics classroom,
- the institutional level, concerning the social norms and interactions within the schools which affect the mathematics teaching in the classrooms,
- the societal level concerning the relationships between mathematics education and institutions in the wider society,

- the cultural level, concerning the relationships between mathematics education and the cultural and historical context of the society.

In this talk I wish to concentrate on the last of these, that is the implications of research at the cultural level, and I will base my talk around a new idea called 'ethnomathematics', which is making mathematics educators think about some very important ideas, such as the following:

- human interactions. Ethnomathematics concerns mathematical activities in society, which take place largely outside school, and it thereby draws attention to the roles which people other than teachers and learners play in mathematics education.
- people and values. Ethnomathematics makes us realise that mathematical activity involves values, beliefs and personal choices.
- interactions between mathematics and languages, because languages act as the principal carrier of many mathematical ideas.
- histories of mathematics. A cultural perspective on mathematics makes us attend to different mathematical histories and to what they tell us about who developed mathematical ideas in different societies.
- cultural roots. Ethnomathematics is making us more aware of the cultural and societal starting points of mathematical development.

In this talk I will focus firstly on the main approaches to researching ethnomathematics which have been used, and illustrate them with particular examples. I will then consider the implications of this research for thinking about mathematics curricula, for mathematics teachers, and for teacher training.

There are three main research approaches being adopted in this area, but it must be kept in mind that although I have separated them out here for illustration and analysis, for some researchers and some societies there is considerable overlap between them.

Mathematics in traditional societies

Firstly there is the research into forms of mathematical knowledge found in traditional societies, where by 'traditional' is meant the kind of society which appears to have been relatively unaffected by modern technological progress. This mathematical knowledge has been explored by researchers working in the anthropological tradition in, for example, Papua New Guinea (Lean, 1993), in Mozambique (Gerdes, 1995), and with the Maori people in New Zealand (Barton and Fairhall, 1995), with Aboriginal Australians (Cooke, 1990) and with the Navajos in N America (Pinxten, 1983).

Much of this research is summarised in Gerdes (1996) and in Barton (1996), and it has produced some fascinating data. For example, did you know that:

- there are more than 2000 different counting systems in Papua New Guinea and Oceania, some use a 5 cycle method, some a 2 cycle. There are many body-counting systems (extensions of finger counting) where the number name is the name of the part of the body being pointed to.
- there are many different ways to add, subtract, multiply and divide (and still get the right answer!)
- there are different ways to find the areas of rectangles. One method used by farmers in Brazil to find the areas of their fields is to find the average lengths of the opposite sides and multiply the averages together.
- there are many different games, puzzles, sports and dances which have mathematical connections
- carpenters, navigators, fishermen, and tailors all have their different mathematical knowledge and skills.

Typical of this research is the study by Glendon Lean (1992) of the counting systems of Papua New Guinea and Oceania. Using a combination of field notes, recorded interviews, secondary source data, and questionnaires completed by students and teachers at universities and schools, he accumulated data about more than 2000 counting systems. He classified the systems according to Salzmann's criteria (Salzmann, 1950) and he was able to dispel the '1-2-many' myth which many people thought was the way that so-called 'primitive' peoples count, and stated " I have been unable to find a single instance of a language which has only the numerals 1 and 2 and which terminates precise enumeration at 2". He certainly found plenty of 2 cycle systems (those which only use the numerals 1 and 2) but in all cases the higher numbers are formed by different combinations of 1 and 2.

The (5,20) cycle systems are also widespread in the region, where the number names for 1 to 5 are associated with the names for the 2 hands and the 2 feet for further counting (hence the 20). These systems are different from the 'body-part tally' systems, where the names of the numbers are the names of the body-parts pointed to. There were also several examples of different 4,5,6 and 10 cycle systems.

This area of research is increasing all the time as other data gets recognised as being of mathematical interest, and as educators in such traditional societies realise the value of preserving their culture rather than just teaching the mathematics of their former colonial masters.

Different histories of mathematics

The second strand of ethnomathematical research comes from the historical research tradition, which is much more established and better known to mathematics educators. This current ethnomathematical development has been stimulated by the realisation that previously published historical analysis had been largely restricted to the Western classical and European traditions. The concerns of the research community are now to document other mathematical histories in different parts of the world. Typical of the new analyses is Joseph's (1991) book "The crest of the peacock - non-European roots of mathematics", written to celebrate the diversity of cultures which have contributed to the rich global store of mathematical ideas.

For example, the cultural history of Iran and of the Muslim world is filled with mathematical ideas, and although much of this tradition will be known by Muslim scholars, it may not be well known by all mathematics educators in Muslim societies. It is certainly not well known by Western mathematics educators, but one can say that it is becoming better known. For example, we have learnt about the importance in mathematical history:

- of the laws of inheritance in Arab societies
- of the designing of mosques and their tiled surfaces
- of locating the direction of Mecca in different parts of the world
- of Arabic astronomy
- of developing geometric proofs for algebraic theorems
- and of the work of mathematicians such as Al-Khwarizmi, Thabit ibn Qurra, Al-Kashi, and Omar Khayyam.

As well as illustrating differences in mathematical aspects such as number systems, and different algorithms for arithmetical calculations, this kind of historical research is demonstrating many interesting similarities such as the fascination with the many ways used to justify, demonstrate or prove the idea now known as the Theorem of Pythagoras. This particular 'theorem' was known to other people including the Chinese (see Ronan, 1981) and many Africans (see Gerdes, 1995) well before Greek mathematicians like Pythagoras became interested in it. Thabit ibn Qurra also developed a generalisation of the Pythagorean theorem which relies on a beautiful and elegant geometric ‘argument’.

Once again as more data comes to light from a variety of historical sources, so ethnomathematicians are able to develop more ideas of significance.

Children’s outside-school mathematical knowledge

The third, and newest of the ethnomathematical research developments, concerns students' outside-school mathematical knowledge. This research has been well summarised by Nunes(1992), who has stimulated much current interest in it by her own work in Brazil with children who buy and sell on the street. Typical of this type of research, but also extending many of the ideas, is the PhD study of Guida de Abreu(1993). Having already documented the ethnomathematics of the sugar-cane farmers of Recife (Abreu and Carraher,1988) using the anthropological approach, she went on to research the ways that their children conceptualised the relationship between the mathematics they learned at school and their 'home' mathematics.

The central question for her research was ' Do the children who succeed at school establish a different relationship with their home mathematics than the children who fail?'. Some of the most important data of the study concerned the strong feelings which the children exhibited about the lack of value which society attached to their fathers' knowledge, which they did not recognise as mathematical. For example:

Interviewer: Why doesn't that man(in a picture) on the tractor know mathematics?
Severina: He doesn't know. He doesn't have a job. He works in sugar-cane.

Later in the interview the student revealed some of her feelings about the way **she** valued her father's knowledge:

Interviewer: Can you tell me what you think about the way your father did the sums, is it the same or different from the way you learned at school?
Severina: It is a different way, he does it in his head, mine is with the pen.
Interviewer: Which do you think is the proper way?
Severina: School
Interviewer: Which do you think gives a correct result?
Severina: My father's.

As well as illustrating the 'disruption', as she called it, caused to children's mathematical education by their beliefs about the relative status of home and school mathematics, Abreu's study also highlighted another phenomenon, the diversity among children in terms of their home mathematics knowledge, which was linked to the extent of their participation in home mathematical practices. The main trend was that the more engaged children were in home mathematics the more likely they were to experience difficulties in school mathematics. This is of course contrary to what one would expect if the educational system was taking the students' outside-school mathematical knowledge into account.

Abreu suggests that one of the most pressing problems arising from studies such as this is "How to organise the school practices in order to minimise the effects of the disruptive relationship between home and school mathematics?"

Having looked at the three main research approaches to ethnomathematics I now wish to focus on the implications of this research.

Implications for the mathematics curriculum

The most immediate and obvious implication from these three research areas concerns the curriculum. This has already started through the development of culturally-based mathematical activities in different countries' systems, related to various mathematical topics mostly in arithmetic, geometry, probability and statistics. However if the challenge of providing a meaningful mathematics education for all children is to be met successfully then a more major curriculum response is going to have to develop.

In particular, there needs to be more consideration of the overall structure of the mathematics curriculum. In general the curriculum which applies in most countries of the world (see Bishop 1993) is not well suited to the introduction of cultural perspectives. It is a curriculum structure which has evolved to suit the mathematical preparation of students for further mathematical study at university. However when we consider the majority of school pupils who will never go on to study mathematics at university, this so-called preparation is shown to be very inappropriate. Furthermore when we look at general educational provision around the world, we can see that inappropriate curricula contribute significantly to the widespread problems of alienation felt by many students about schooling in general, which make many leave school early.

I have already proposed such a curriculum structure (Bishop, 1991) based around the six fundamental activities which all cultures and societies have been shown to develop:

- Counting: This is the activity concerned with the question "How many?" in all its forms and variants eg. there are many ways of counting and of doing numerical calculations. We can recognise here the mental abilities of numerical reasoning, mental calculation, quantitative reasoning, and numerical reckoning (see for example, Starkey, 1992). The mathematical ideas derived from this activity are numbers, calculation methods, number systems, number patterns, numerical methods, statistics, etc.

- Locating: This activity concerns finding your way in the structured spatial world of today, with navigating, orienting oneself and other objects, and with describing where things are in relation to one another. We use various forms of description including maps, figures, charts, diagrams and coordinate systems. This area of activity is the 'geographical' aspect of mathematics. The mental abilities of spatial orientation, and coordination, and the use of kinaesthetic and other kinds of imagery are particularly important here (see for example, Tartre, 1990). The mathematical topics derived from this activity are: dimensions, Cartesian and polar coordinates, axes, networks, loci, etc.

- Measuring: "How much?' is a question asked and answered in every society, whether the amounts which are valued are cloth, food, land, money or time. The techniques of measuring, with all the different units involved, become more complex as the society increases in complexity. Measuring involves some of the same mental abilities as does Counting but it also develops those of estimating, approximating, and evaluating (see for example, Silver, 1994) Mathematical topics derived here are: order, size, units, measure systems, conversion of units, accuracy, continuous quantities, etc.

- Designing: Shapes are very important in the study of geometry and they appear to derive from designing objects to serve different purposes. Here we are particularly interested in how different shapes are constructed, in analysing their various properties, and in investigating the ways they relate to each other. The mental abilities which are developed by these activities include visualisation and imagination, figural interpretation, drawing and other forms of representing (see for example, Presmeg, 1986). The mathematical topics derived are: shapes, regularity, congruence, similarity, drawing constructions, geometrical properties, etc.

- Playing: Everyone enjoys playing and most people take playing very seriously. Not all play is important from a mathematical viewpoint, but puzzles, logical paradoxes, some games, and gambling all involve the mathematical nature of many activities in this category. From the perspective of mental abilities, some of the earlier ones mentioned are also important here, but playing seems to develop particular skills of strategic thinking, guessing, and planning (see for example, Brady, 1978) Mathematical ideas derived here are: rules, procedures, plans, strategies, models, game theory, etc.

- Explaining: Trying to explain to oneself and to others why things happen the way they do is a universal human activity. In mathematics we are interested in, for example, why number calculations work, and in which situations, why certain geometric shapes do or do not fit together, why one algebraic result leads to another, and with different ways of symbolising these relationships. This activity of explaining involves many of the earlier mental abilities, but particularly develops logical reasoning, and also verbal reasoning (see for example, Hersh, 1993) Mathematical topics derived here are: logic rules, proof, graphs, equations etc.

This structure enables many culturally-relevant activities from the wider society to be used in class as well as encouraging the development of more generalised mathematical ideas. The table below shows how the six activities map onto the six parallel mathematics curriculum strands which are followed in Victoria, Australia:

	Space	**Number**	**Measure-ment**	**Chance and data**	**Algebra**	**Mathe-matical tools**
Counting		✓	✓	✓	✓	✓
Locating	✓		✓			✓
Measuring	✓		✓			✓
Designing	✓					✓
Playing				✓		✓
Explaining	✓	✓	✓	✓	✓	✓

While there is a certain level of agreement between the two classifications, and therefore it is possible to build in some culturally relevant activities, it is also clear that some of the activities are currently underepresented eg. locating, measuring, designing and playing. I and my colleagues are trying to increase their influence in the curriculum.

I firmly believe that if more experimentation were undertaken with socially and culturally-relevant mathematics curricula, then more teachers would be able to use culturally-based mathematical activities in their classrooms and more young people would feel that they were studying a more meaningful mathematical education.

Implications for teachers

As well as having implications for the mathematics curriculum, ethnomathematical research also suggests other important aspects of teaching need to change. Firstly consider the role of the teacher. Till now the mathematics teacher's focus has been largely on the transmission and explication of conceptual and procedural mathematical ideas. As we have seen, however, developments in ethnomathematics have indicated that teachers need to attend also to societal and cultural factors.

In particular, and among other things, teachers:

- should recognise the crucial importance of their role as the fundamental legitimiser of knowledge in the classroom, helping to determine how different mathematical ideas are valued and accepted by the pupils,

- should try to find out as much as possible about the pupils, particularly about their outside-school mathematical knowledge,
- should continually provide opportunities through classroom activities to allow pupils to bring their mathematical knowledge explicitly into the learning activities,
- should be continually encouraging pupils to describe their knowledge, to check it with their peers, to symbolise it in different ways, to explain its context, and to relate it, as they see it, to the mathematical tasks at hand
- should be continually aware of, and should explain to the pupils, their own values in relation to the mathematical knowledge being discussed.

What this means that, in the primary school and in the early part of secondary school, the classroom activities and materials should relate as closely as possible to the pupils' out-of-class experiences. The problems should be set in familiar contexts, the examples should be from the pupils' environment, the language used should be familiar to the pupils, the numerical data should be close to the pupils' experience etc. Later on, as the pupils become familiar with the strategies, concepts and routines of mathematical activity the teacher can introduce problems, language and situations from outside the pupils' experience. Unless the preparatory has been done, however, the learning will probably become increasingly meaningless, leading to all the familiar problems described earlier.

Implications for teacher training.

One of the important tasks for teacher educators in mathematics is how to help future teachers understand their role as 'mathematical enculturators' (Bishop, 1991). This means educating them about the cultural aspects of mathematics, about values in mathematics, and about different histories of mathematics. The six categories of activities can be very helpful in designing a course to introduce student teachers to mathematical knowledge in different cultures, with references to some of the research literature.

For example, it could be appropriate to explore different cycles of finger counting, or different number symbols, and number pattern work with different multiplication algorithms could be carried out (see, for example, Joseph, 1992). Similarly, designing activities from different cultures can easily be incorporated into a mathematics course, and mathematical games from different cultures can also be used, with magic squares, number combination games, games of chance and strategy (see Ascher, 1981, Bell and Cornelius, 1988, and Zaslavsky, 1973).

Often a particular piece of anthropological data can be used to create interesting mathematical activities for school lessons, which can then be demonstrated and discussed in teacher-training courses. For example, Gerdes (1988) reports that in Mozambique, certain rural house builders use four pieces of rope tied together to lay out a rectangle, which is the shape of the house. The four pieces of rope are the same length and tied together at one end. This situation can be used as an introduction to some geometrical features of rectangles, squares and other quadrilaterals:

> Give the students some string tied together as indicated. How can you lay out a rectangle?
> Can you make different rectangles?
> How do you know when you have a rectangle?
> What angles are the same?
> What other shapes can you make with these strings?
> What other angles are important?

The situation can be investigated further:

(a) Suppose the strings are not all equal, what shapes can you now make?

(b) Suppose you have five strings all the same length. What shapes can you now make?
(c) What other polygons could be made this way?

Other mathematical investigations can very easily be stimulated by mathematical activities from different cultures. All of these develop in the future teachers important ideas about the multi-cultural nature of mathematics, which will enable them to be prepared for doing these activities with the pupils in the schools. Here are some other stimulating starting points:

Body-counting and finger-counting methods.
Circular calendars
Rug-weaving patterns
Basket weaving methods
The Quipu (a South American system for recording numerical information using string)
Calculations on the abacus
Sun-dials
String games
Body measures (cubits etc)
Numerological and astrological prediction
Board game analyses
Magic squares

However, there are several principles which are important when using these ideas in teacher-training courses:

- Keep as much as possible of the cultural <u>context</u> involved with the activity. Usually the meaning and significance of a particular activity is given by its context.
- Try to include material resources like rugs, baskets, counting frames, in the course as well as written material. Mathematical ideas are represented in cultures in many ways.
- Collect, and use, pictures of mathematically interesting objects from other cultures if you can't get the objects, or if they are too big (such as a house or a temple).
- Try to inject these ideas into every mathematical course.

Another very important principle concerns putting 'people' back into the mathematics education curriculum. Often we are so concerned with the mathematical ideas themselves that we lose sight of the fact that these ideas are developed by people, and this is their connection with culture. Another point is that the student-teachers are not merely recipients of cultural knowledge, they are recreators and reconstructors of it. Each generation must in some way relive earlier experiences to ensure that the ideas are live ideas contextualised within today's society rather than dead knowledge merely passively received and quickly forgotten.

These then are some of the ways in which the mathematical curriculum in the teacher training courses can demonstrate the ideas of culture in mathematics. Student teachers need the opportunity to reflect on how mathematical ideas have developed in different histories, in order for them to have a clearer idea of how they can fulfil their crucial role as mathematical enculturators - the role of introducing children to the rich culture of mathematics.

Furthermore if all mathematics educators took these cultural ideas seriously, I am certain that mathematics education would be not a meaningless and boring set of routines for many pupils, as it is at present, but a satisfying, interesting, and meaningful experience.

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